The Quest Towards Analytical Solutions of Linked Fault Tree Models using Binary Decision Diagrams(*)

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- Motivation and issues with current PSA softwares
- Binary Decision Diagrams (BDD) as an alternative
- Research and development at KKL and ETH Zurich
- Insights and outlook
- Presentation of the *NeuralSpectrum* Software

(*) From the PhD thesis:

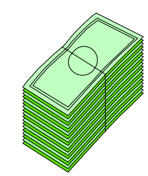
"Analytical Solutions of Linked Fault Tree Probabilistic Risk Assessments using Binary Decision Diagrams with Emphasis on Nuclear Safety Applications"

Motivation and issues with current PSA softwares

- **The (Swiss) NPP have to submit best-estimate, plant-specific PSA models to the regulatory authority**
  - Level 1 PSA: Calculation of the Core Damage Frequency (CDF)
  - Level 2 PSA: Calculation of the Plant Damage State (PDS) frequencies and associated radiological consequences
  - For internal events, area events and external events
  - Typically, the PSA modeling techniques are based on the Fault Tree / Event Tree approach (FTA)

Motivation and issues with current PSA softwares

- **Risk Informed Applications**
  - Evaluation and support of Plant / TechSpec Modifications (e.g. relaxation)
  - Risk Informed In Service Inspection
  - Aging Programs
  - ILRT frequency relaxation
  - Outage optimization



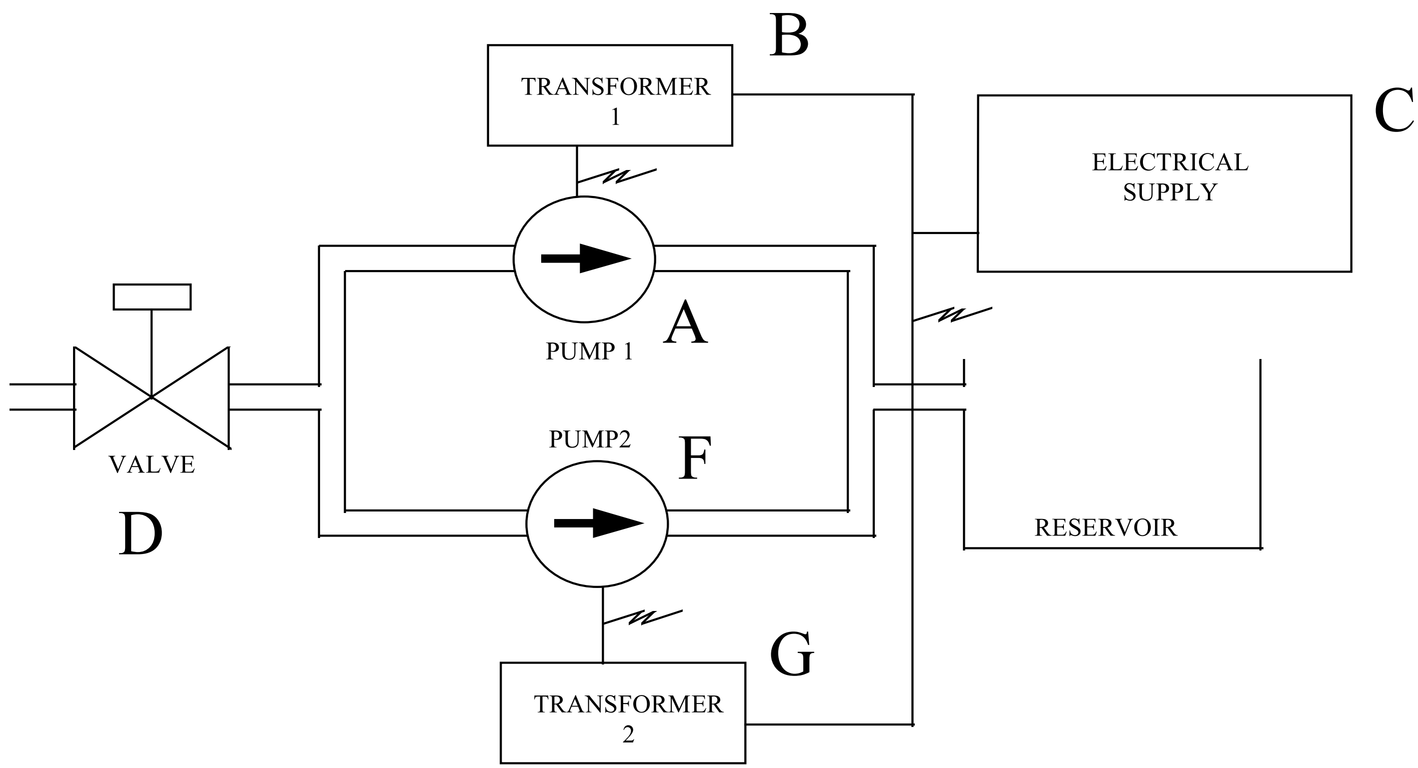
Motivation and issues with current PSA softwares

- **Worldwide Fault Tree Analyses (FTA) are performed with codes that produce wrong results**
  - Rare event approximation…
  - … but not only !
- **Resulting risk Importance measures are even more wrong (conservative or optimistic, no one can know)**

$$RIF_{x_i} = \frac{CDF(x_i = 1)\nearrow}{CDF \nearrow}$$

Motivation and issues with current PSA softwares

- **What is the mean unreliability of the system’s function, based on individual Basic Events probabilities ?**

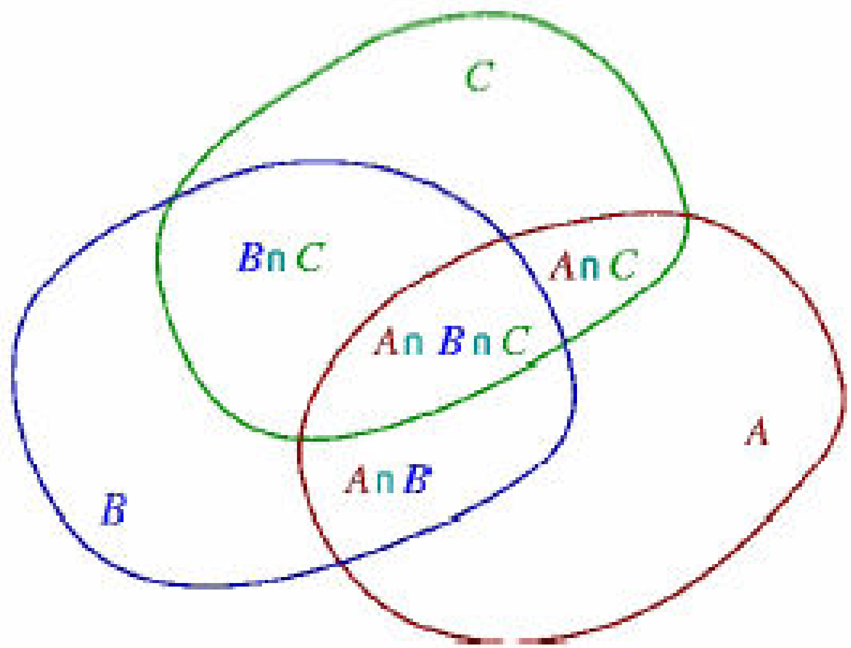


$$P_{top} = D + C + AF + AG + BF + BG$$

Motivation and issues with current PSA softwares

- **The rare event approximation (Moivre‘s equation)**

$$|A_1 \cup \ldots \cup A_p| = \sum_{1 \le i \le p} |A_i| - \sum_{1 \le i_1 < i_2 \le p} |A_{i_1} \cap A_{i_2}| + \ldots + (-1)^{p-1} |A_1 \cap \ldots \cap A_p|$$

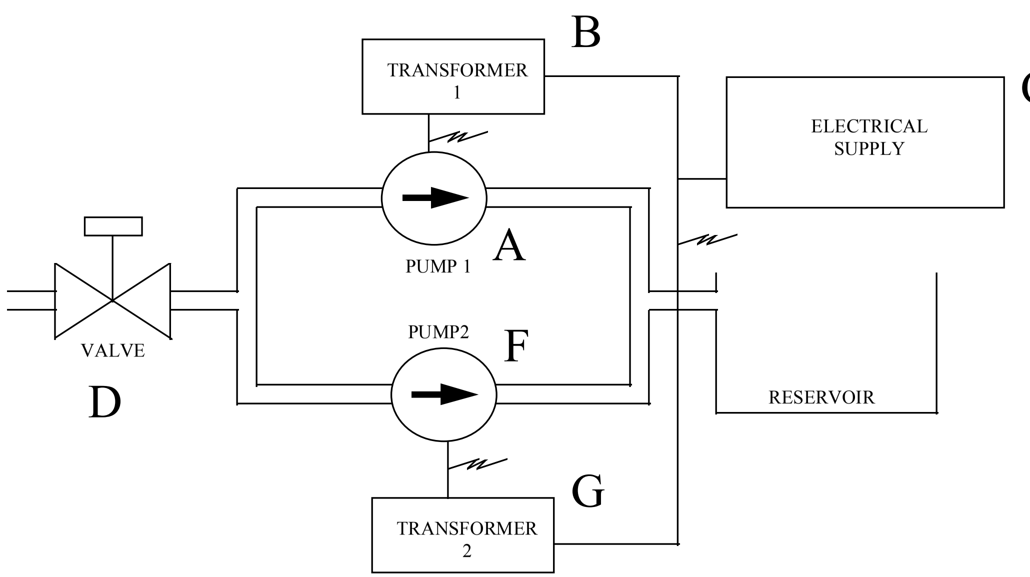


Motivation and issues with current PSA softwares

- **Analytically correct result yields:**

$$P_{top} = (d + f + g + c - d f - d g - d c - f g - f c - g c + d f g + d f c + d g c + f g c - d f g c)(a + b + c + d - a b - a c - a d - b c - b d - c d + a b c + a b d + a c d + b c d - a b c d)$$

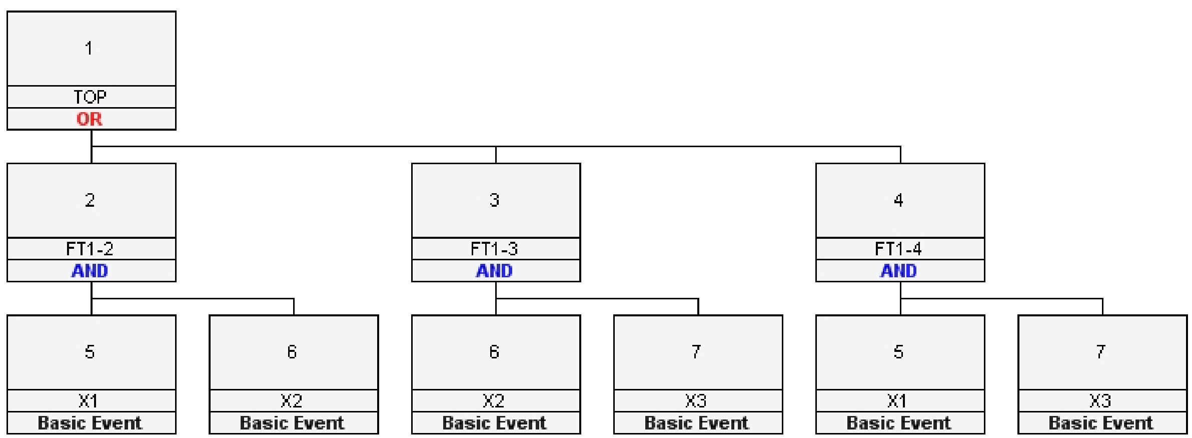
$$= [c - a(-1 + b)(-1 + c)(-1 + d) + b(-1 + c)(-1 + d) + d - c d] \cdot [f - c(-1 + d)(-1 + f)(-1 + g) + d(-1 + f)(-1 + g) + g - f g]$$



Motivation and issues with current PSA softwares

- **Consider a « 2 out of 3 system » given by the Boolean equation:**

$$T = (x_1 \wedge x_2) \vee (x_1 \wedge x_3) \vee (x_2 \wedge x_3) \qquad p(x_1) = p(x_2) = p(x_3) := q$$



$$p(T) \neq p(x_1)p(x_2) + p(x_1)p(x_3) + p(x_2)p(x_3) = 3 \cdot q^2$$

$$p(T) = p(x_1)p(x_2) + p(x_1)\underbrace{(1-p(x_2))}_{Success}p(x_3) + \underbrace{(1-p(x_1))}_{Success}p(x_2)p(x_3) = 3q^2 - 2q^3$$

Motivation and issues with current PSA softwares

- **Other issues include:**
  - Wrong treatment of negative logic
    (e.g. forbidden maintenance unavailabilities according to TechSpec)
  - Quantification cutoff (typically 1E-12 to 1E-14)
  - Wrong interpretation of risk importance measures of components, systems and safety divisions (RIF, FV, etc.)
  - Treatment of exchange events
- **Advanced PSA models include HRA, CCF, seismic and phenomenological events, where failure probabilities approach 1**
- **It is accepted that current quantification tools have reached their own limits [Rauzy, 2001]**

Motivation and issues with current PSA softwares

- **Develop a new PSA quantification methodology that:**
  - Overcomes the deficiencies of the rare approximation, i.e. credit success branches, calculate the rare event up to **infinite order**
  - Yields a correct evaluation of Risk Importance Factors (RIFs)
  - Support the treatment of negative logic
  - Do not apply cutoff when generating the sequences
  - Improve calculation speed and result consistency

Binary Decision Diagrams (BDD) as an alternative

- **Shannon expansion**

$$x \rightarrow y_0, y_1 := (x \wedge y_0) \vee (\overline{x} \wedge y_1) := ite(x, y_0, y_1)$$

- **Shannon expansion of *t* with respect to *x***

$$t = x \rightarrow t[1/x], t[0/x] \Rightarrow t = (x \wedge t[1/x]) \vee (\overline{x} \wedge t[0/x])$$

  - *t[0/x]* and *t[1/x]* both contain one less variable than the expression *t*
  - We can recursively find ITEs up to the basic elements *0* (false) and *1* (true)

Binary Decision Diagrams (BDD) as an alternative

- **Example for the „2 out of 3" system** $t = AB + BC + AC$

$t = A \rightarrow (t_0, t_1)$

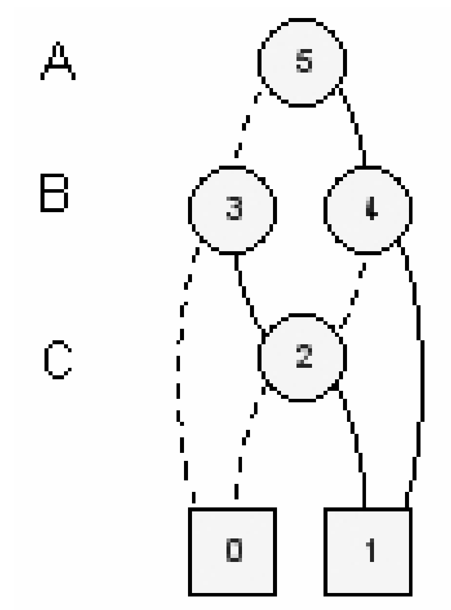
☼ $t_0 = B \rightarrow (0, t_{01})$

☼ $t_1 = B \rightarrow (1, t_{10})$

☼ $t_{01} = C \rightarrow (1, 0)$

☼ $t_{10} = C \rightarrow (1, 0)$

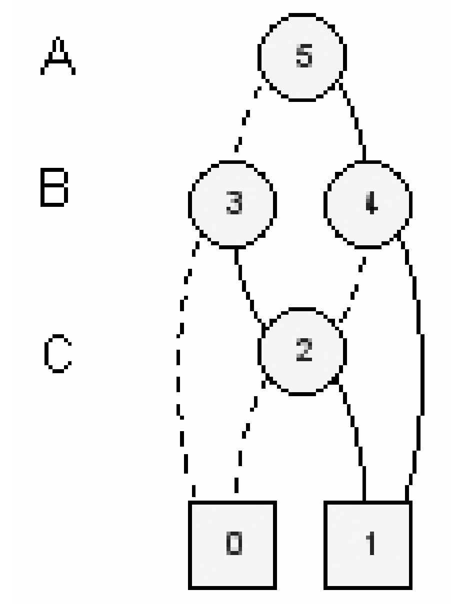
=> $t = A \rightarrow (B \rightarrow (0, C \rightarrow (1, 0)), B \rightarrow (1, C \rightarrow (1, 0)))$



Binary Decision Diagrams (BDD) as an alternative

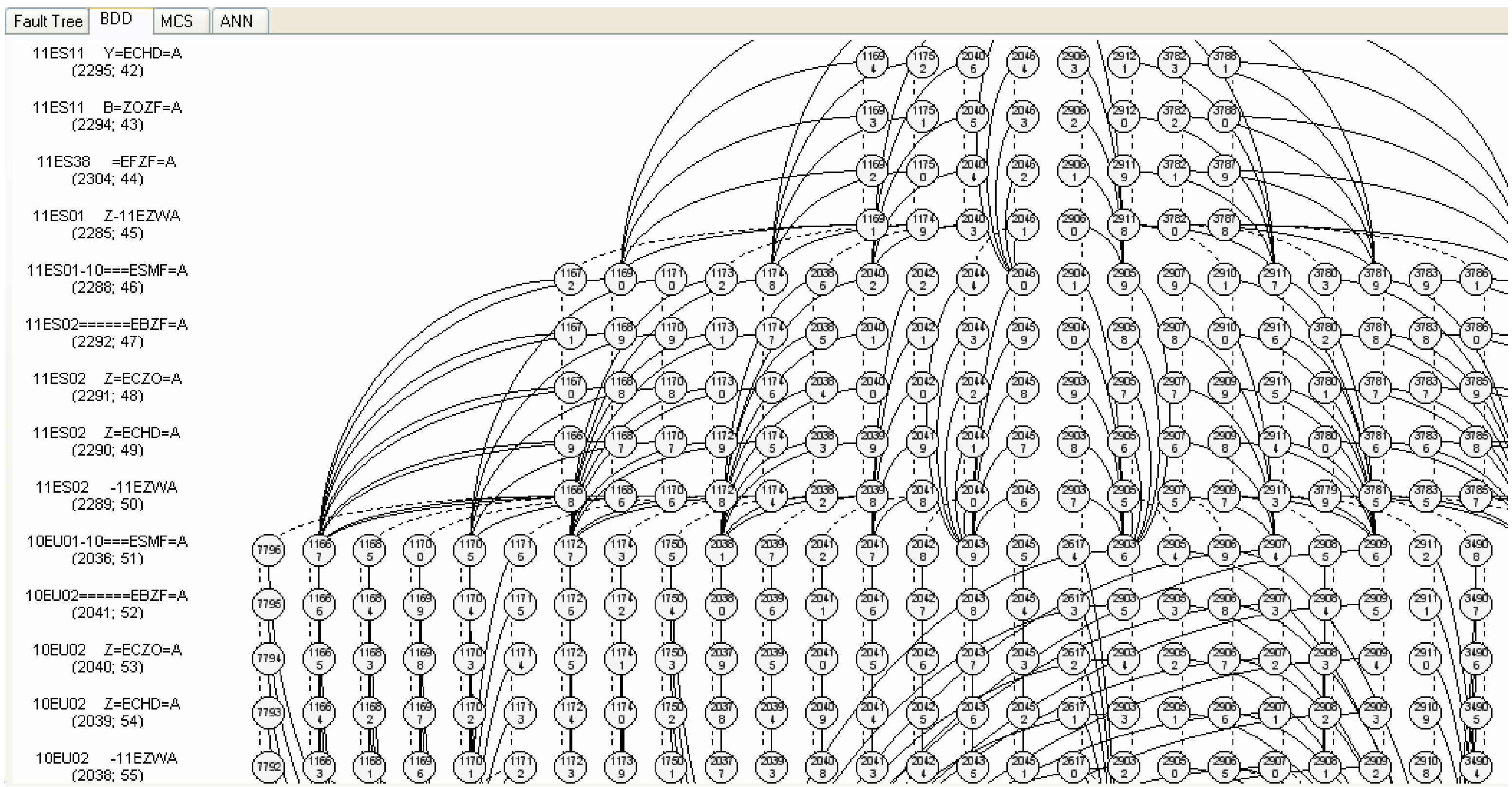
- <u>**Canonical formulation of Boolean equations !**</u>

- **Example:** $P_{t=true} = AB + A\,(1-B)\,C + (1-A)\,B\,C$



Binary Decision Diagrams (BDD) as an alternative

- BDD complexity is not related to the number of prime implicants of the encoded formula
- This small BDD (37620 nodes) encodes a total of 10^9 cutsets

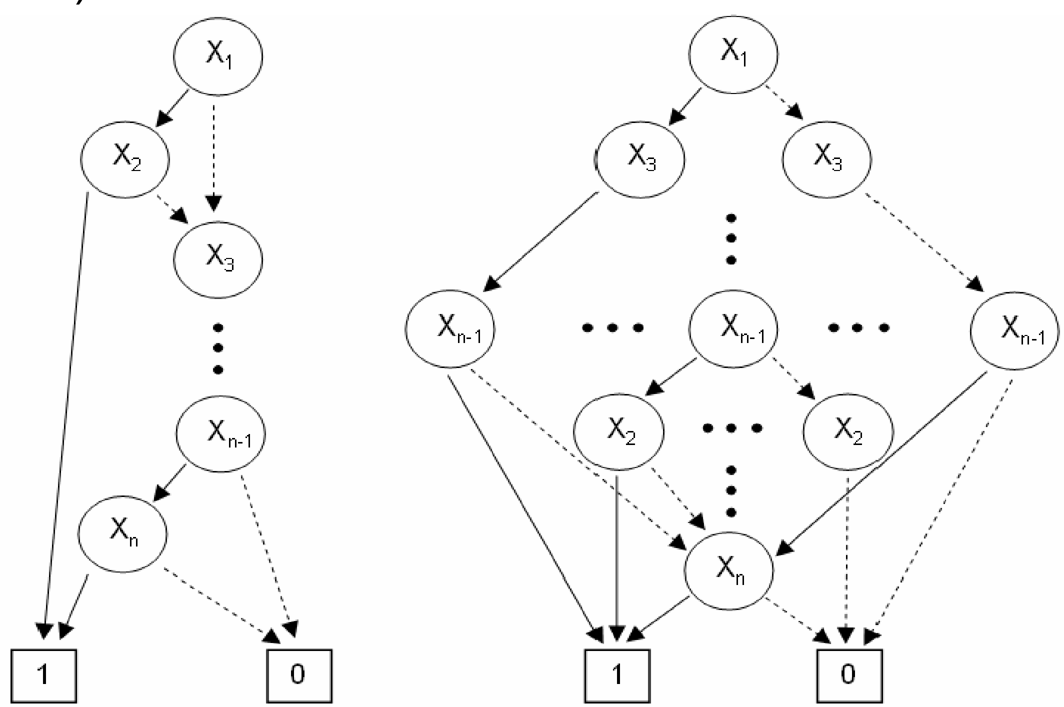


Binary Decision Diagrams (BDD) as an alternative

- **Let’s have a closer look at it !**
  - HPCS System of the Leibstadt NPP

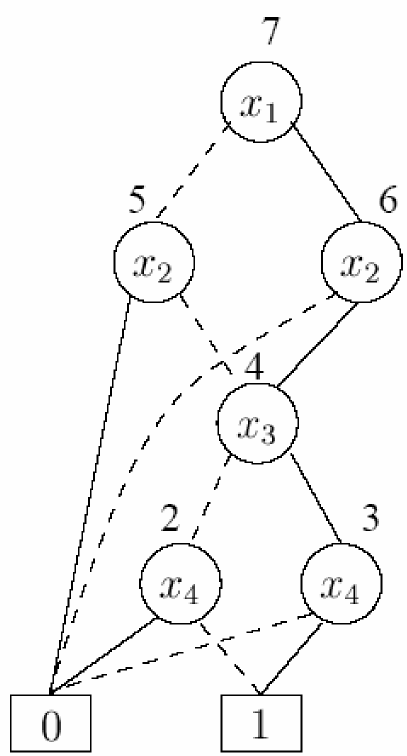
Binary Decision Diagrams (BDD) as an alternative

- **Impact of variable order on BDD size**
  - ☼ From linear ☺ to exponential ☹
  - ☼ ☹ Finding the best order is of **NP-Complete complexity** (Bollig / Wegener, 1996)



Binary Decision Diagrams (BDD) as an alternative

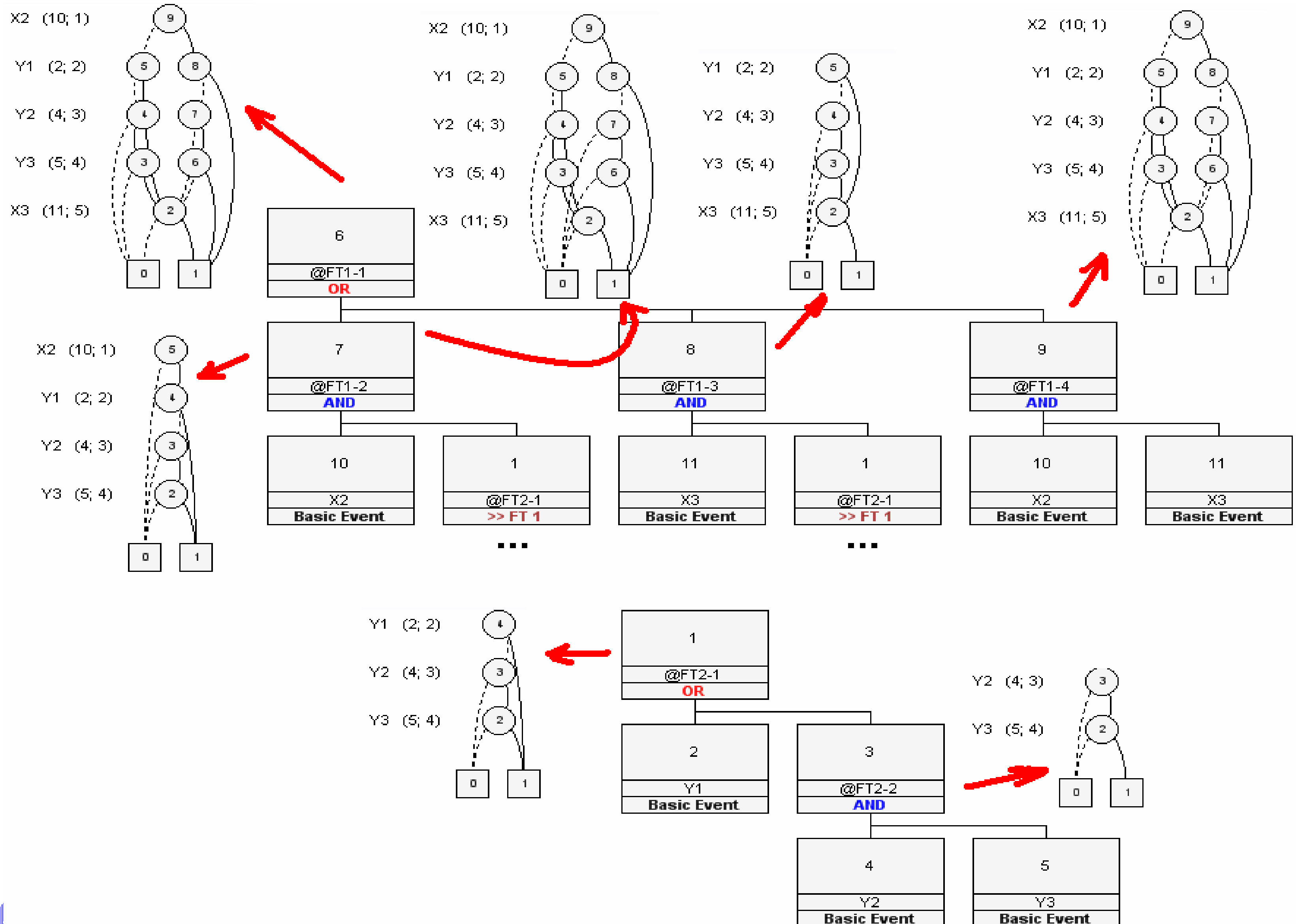
- **Computer encoding of BDDs**
  - Open-ended node table (dynamic)
  - Open-ended Hashtable (e.g. unique table)



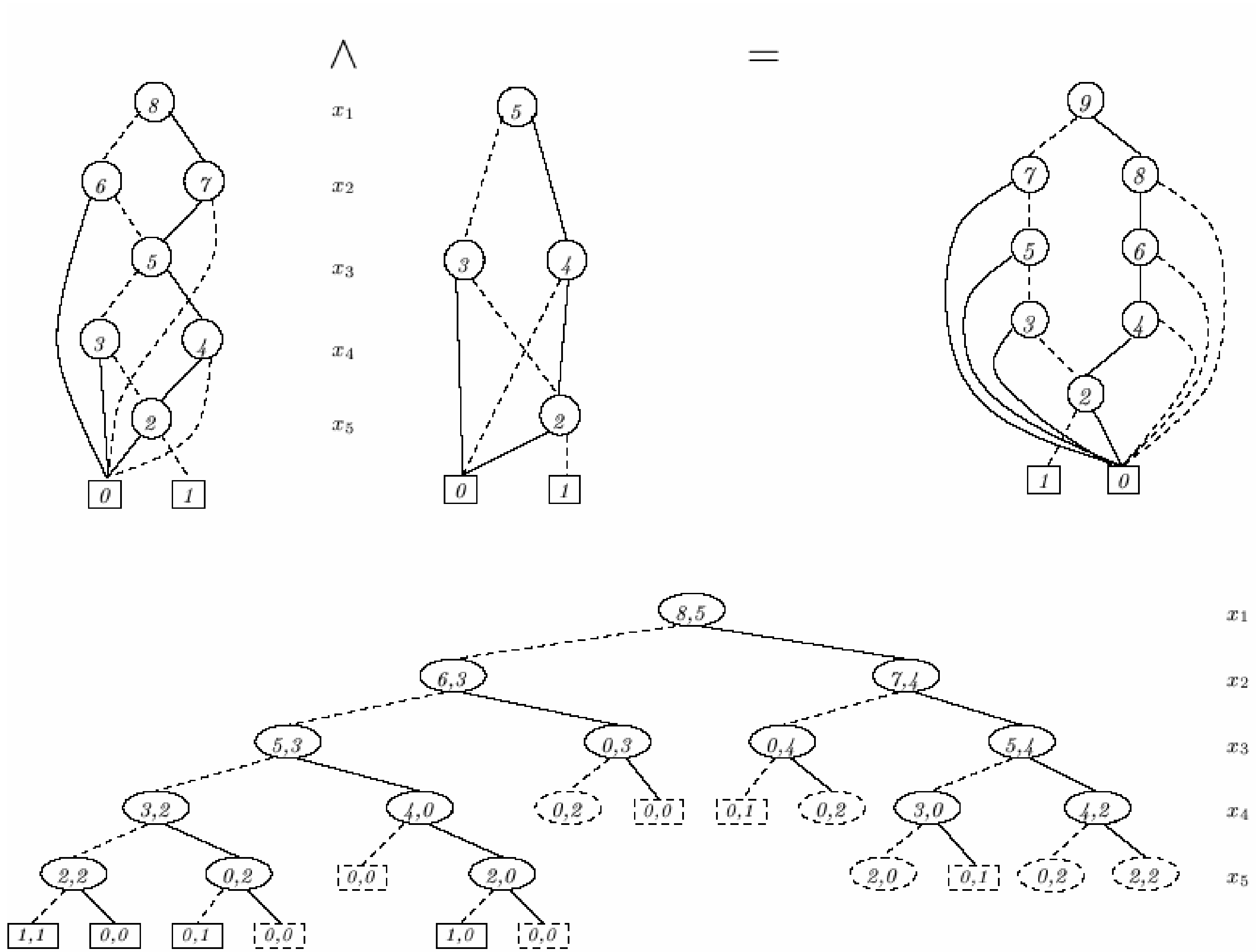
$T : u \mapsto (i, l, h)$

u	var	low	$high$
0	5		
1	5		
2	4	1	0
3	4	0	1
4	3	2	3
5	2	4	0
6	2	0	4
7	1	5	6

Binary Decision Diagrams (BDD) as an alternative



Binary Decision Diagrams (BDD) as an alternative



Binary Decision Diagrams (BDD) as an alternative

Algorithm $Apply[T, H](op, u_1, u_2)$

Require: u_1 and u_2 the top nodes of the BDD to assemble.
Ensure: The resulting BDD.

if $G(u_1, u_2) \neq \emptyset$ **then**
    return $G(u_1, u_2)$
else if $u_1 \in \{0, 1\}$ and $u_2 \in \{0, 1\}$ **then**
    $u = op(u_1, u_2)$
else if $var(u_1) = var(u_2)$ **then**
    $u = newnode(var(u_1), apply(low(u_1), low(u_2)), apply(high(u_1), high(u_2)))$
else if $var(u_1) < var(u_2)$ **then**
    $u = newnode(var(u_1), apply(low(u1), u2), apply(high(u_1), u2))$
else
    $u = newnode(var(u_2), apply(u_1, low(u_2)), apply(u_1, high(u_2)))$
end if
$G(u_1, u_2) \leftarrow u$ {Add to computation table}
return u {Returns node index}

Binary Decision Diagrams (BDD) as an alternative

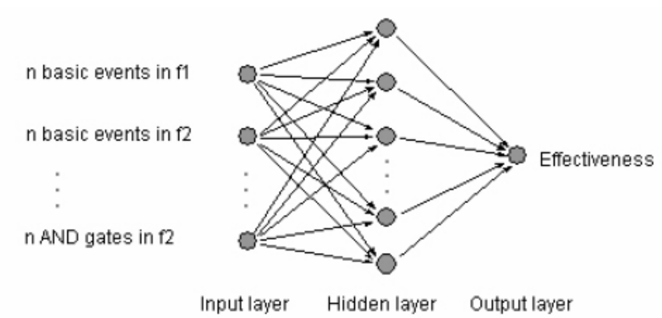
- **Previous Work**
  - BDD have been implemented in the early 90‘s for Integrated Circuits (IC) checking and IC optimization (16 and 32 bits)
  - Some attempts to convert small to medium size models (typically with a few hundreds Basic Events) have succeeded
  - All attempts with more Basic Events (>1000) have failed due to the exponential growth in complexity (BDD blow up)

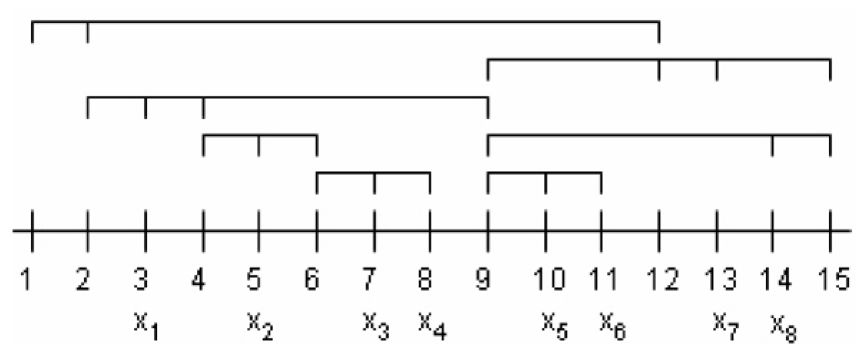
Research and development at KKL and ETH Zurich

- **Development of Fault Tree to BDD conversion engine**
- **Development of statistical measures**
- **Analysis of Fault Tree model pre-processing (rewriting) techniques**
  - Basic Event occurrence based ordering
  - Weights fan-out pre-processing →

$$W(v) = \begin{cases} 1 & \text{for basic events} \\ \sum_i W(v_i) & \text{for gates} \end{cases}$$

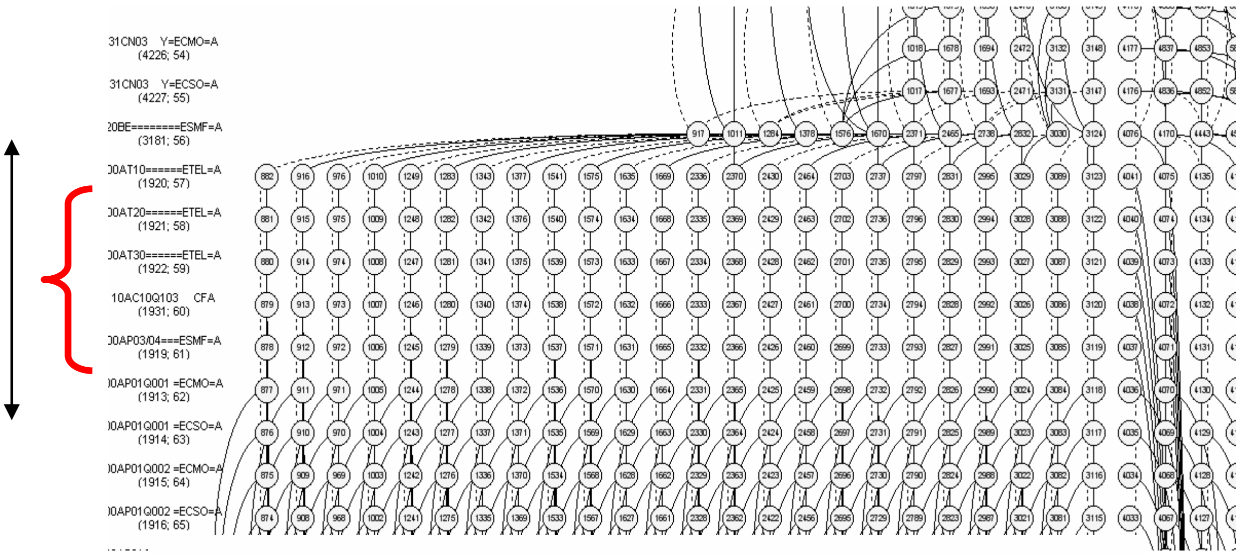
  - Hypergraph optimization techniques
  - Artificial Neural Network





Research and development at KKL and ETH Zurich

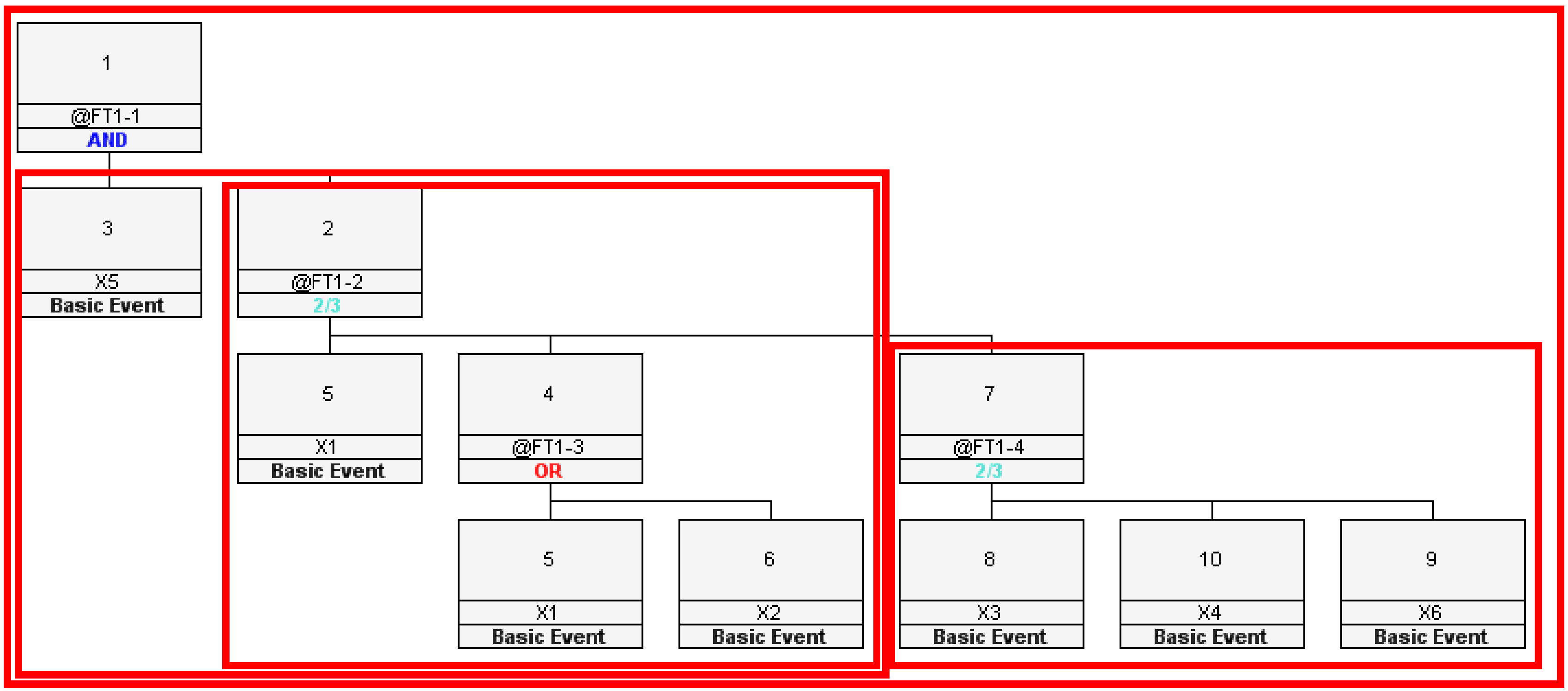
- **Development of dynamic optimization techniques (e.g. Sifting, p-cut variable arrangement)**
- **Development of Group-Sifting for FTA**



	DFLM	Regular sifting	Group-sifting
HPCS	6545	3204	497
LPCS	206'503	40'656	7763
RHR/A	306'339	99'945	11'948

Research and development at KKL and ETH Zurich

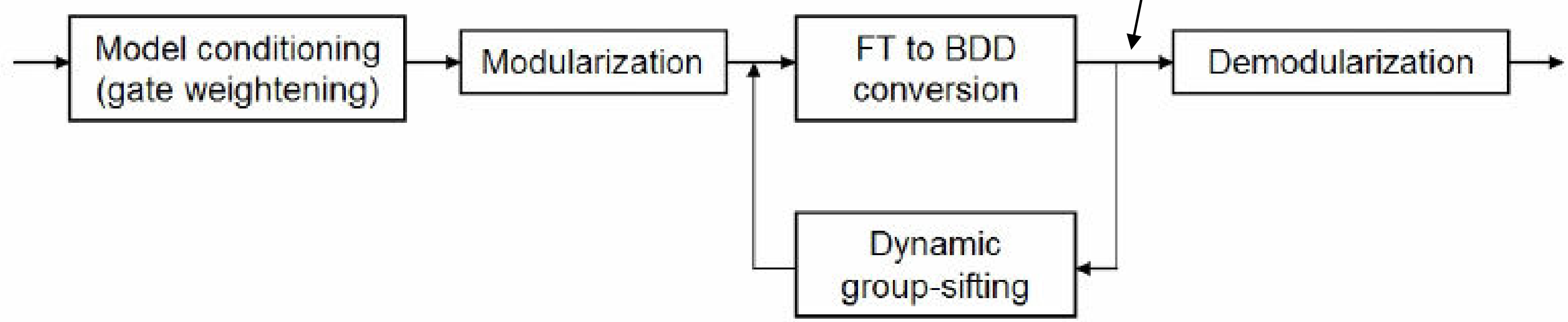
- **Development and analysis of modularization techniques**
  - Occurrence vectors and detection criteria



Research and development at KKL and ETH Zurich

- **Dynamic Group-Sifting using modularization (*DGSM*)**

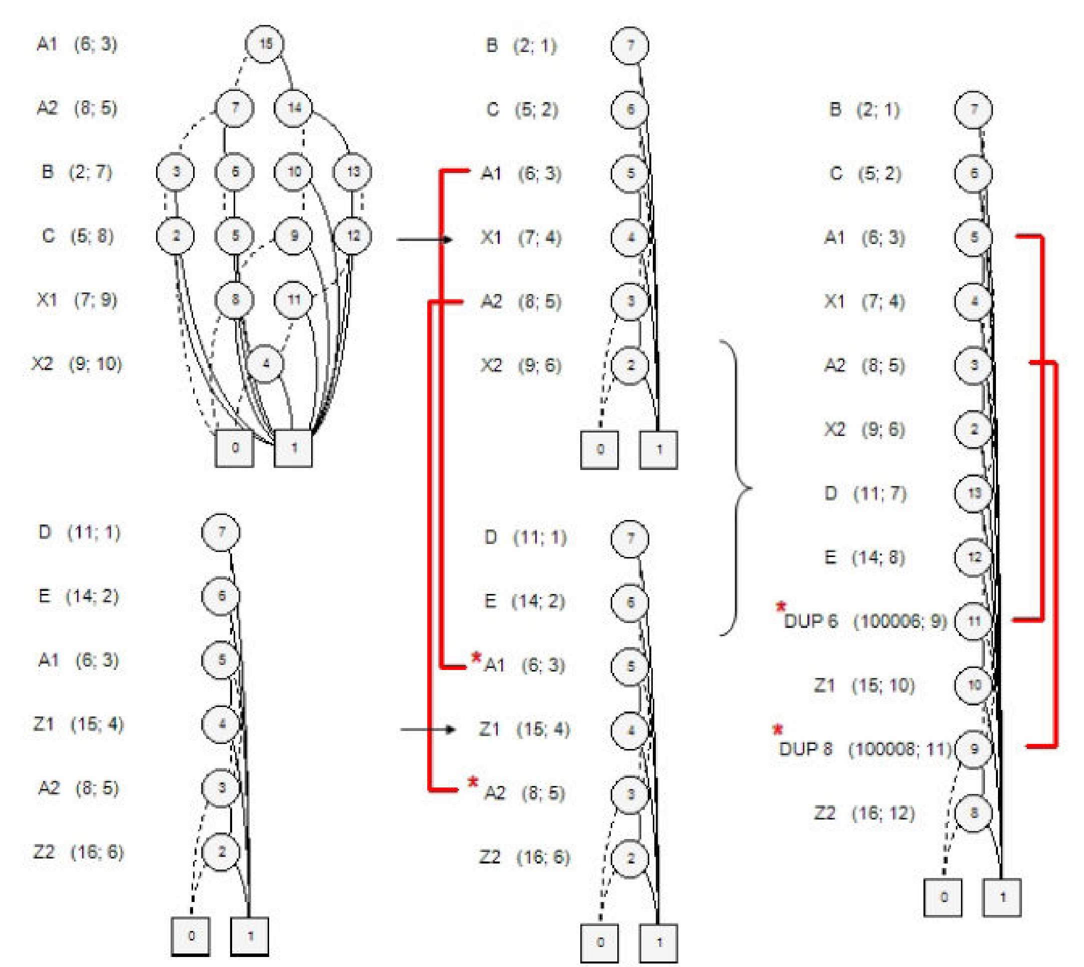
$$size_{BDD_R} > \left(\left(\frac{size_{BDD_R}}{N} \right)^{\alpha} + 1 \right) \times \left(size_{BDD1} + size_{BDD_2} \right)$$



Systems	DFLM	Dynamic group-sifting	Dynamic group-sifting using modularization
HPCS	6545	761	497
LPCS	206’503	3050	3053
RHR/A	306’339	3117	3117
RHR/B	impossible	52’447	21’177
ECCS	impossible	impossible	1’781’100 (CPU= 11 hours)

Research and development at KKL and ETH Zurich

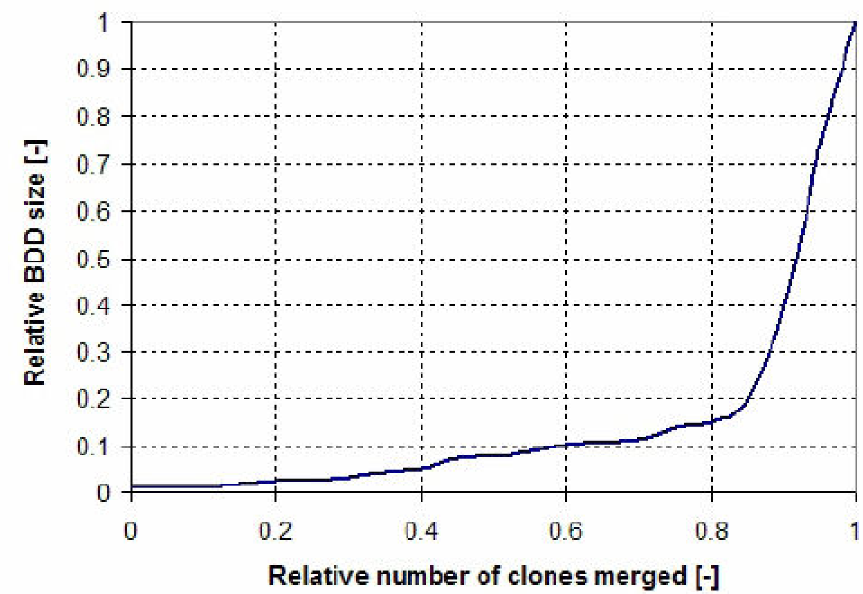
- **Algorithm *FUSION* (BDD as objects)**

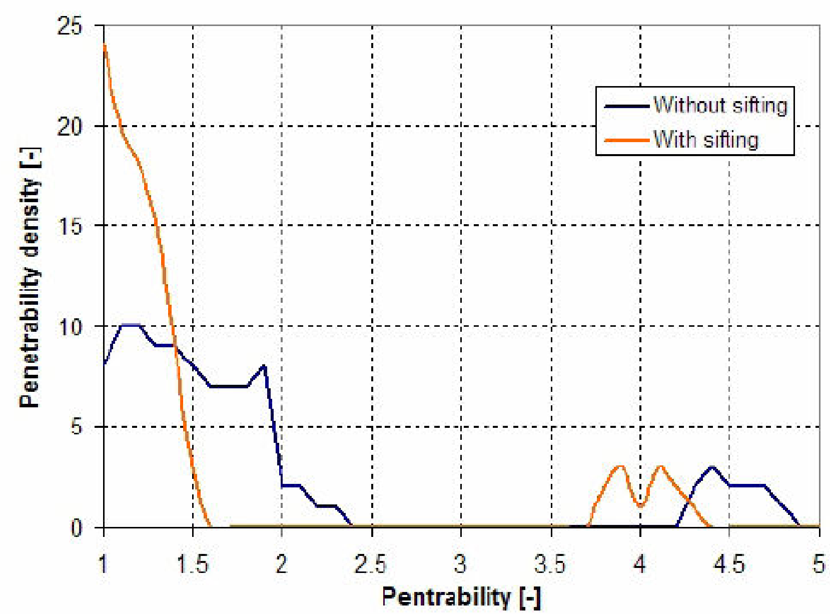


Research and development at KKL and ETH Zurich

- **Performance of *FUSION***
  - About 90% of the variables can be merged without major impact on BDD size
  - Penetrability *p*: effective identification of mathematical “hot spots”

$$p(x) = \frac{Size(Opt(BDD_{x\ merged}))}{Size(BDD)}$$





Research and development at KKL and ETH Zurich

- **Improved Dynamic Group-Sifting Using Modularization (*IDGSM*)**
  - Further limitation of global perturbation when optimizing locally
  - Online identification and treatment of “hot spots” using penetrability spectrum
  - Improvement in the Group-Sifting algorithm
  - Use of genetic optimization algorithms
  - Generation and treatment of “clones” (Algorithm ***FUSION***)

Insights and outlook

- **Results and insights:**
  - FTA quantification using BDD requires complex algorithms and programming techniques
  - The combination of global, static, dynamic and BDD objects techniques proved to be effective when dealing with large models
  - We succeeded in converting the Leibstadt PSA model to a BDD form of more than 1’500’000 with 30 clones, for a total of about 3500 basic events
  - The BDD covers a complete event tree sequence that includes reactor shutdown and reactor cooling with the eight Emergency Core Cooling Systems of the Leibstadt Nuclear Power Plant (including support systems)

Insights and outlook

- **The nuclear industry is facing a major issue (and is not yet fully aware of it):**
  - Worldwide probabilistic analyses are performed with codes that produce wrong results (conservative or optimistic, no one can know)
  - New IAEA requirements are impossible to address with existing FTA quantifiers (e.g. seismic assessments)
  - Utilities and authorities *still* trust the results of existing FTA quantifiers, applying approximations where they should not be applied
    (→ blind-trust in numbers)
  - The techniques developed in this study raises the BDD approach to a mature technology for PSA model solving